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REVIEW

Dynamics of Polymeric Liquids. Volume 1. Fluid Mechanics. By R. B. Bird, R. C. Armstrong and O. Hassager. 470 pp. £22.00. **Volume 2. Kinetic Theory.** By R. B. Bird, O. Hassager, R. C. Armstrong and C. F. Curtiss. 257 pp. £20.00. Wiley, 1977.

This two-volume text is likely to become a valued member of any well-stocked library covering fluid mechanics. It is written largely by chemical engineers, and is in the style of that now classic text *Transport Phenomena* by Bird, Stewart & Lightfoot, although some will recall an even earlier antecedent in *Molecular Theory of Gases and Liquids* by Hirschfelder, Curtiss & Bird. The present authors rightly call their work an introductory textbook: that is not to say that it is a simplified, discursive, or limited account aimed only at those seeking a general impression of the basic ideas; rather that it is meticulously written, compendious, explanatory account aimed at the highly motivated student or worker anxious to study in detail some of the techniques and results generally accepted as important in this growing field of investigation. Large numbers of worked and unworked examples and questions for discussion are given throughout.

It is refreshing to come across one of those rare texts that represent the result of a determined belief in the importance of setting out all the details of a new theoretical development with the same pedagogical care as is usually only lavished on the trivial. Contrary to many contemporary predictions, such texts often remain in print long after the focus of attention in the subject concerned has shifted to other matters, and when the results originally reported have become commonplace rather than new and exciting. Lamb's *Hydrodynamics* is one such. The secret of their success lies in their not assuming too much of the reader, in consciously avoiding intellectual arrogance, in worrying out all the little paradoxes and difficulties that many of us conceal or slur over, and in the authors being in the forefront of research.

The Dynamics of Polymeric Liquids has grown out of a long, stable, unhurried and successful involvement in the subject by a large number of workers at the University of Winsconsin, whose roots have covered the mathematical, physical, chemical and engineering aspects. There is therefore an underlying balance about the whole approach, even though the development given is specific and limited. The authors have deliberately restricted their attention to polymeric fluids, and then mainly to dilute solutions. Nevertheless, this apparently small area of interest covers a good part of the experimental and theoretical work that has been done on non-Newtonian elastic liquids, and so the book forms a good introduction to non-Newtonian fluid mechanics in general.

Volume 1 concentrates on the fluid mechanics of viscoelastic fluids. Except in chapter 2 (on the structure of polymeric fluids), the fluids are treated as homogeneous continua. Although non-isothermal flow problems are treated to some extent, this is only done in the context of purely viscous fluid models, so the interaction between the energy equation and the other equations of motion is relatively simple.

Chapter 1 presents a review of Newtonian fluid mechanics that prepares the reader for the type of flow problems that the authors have in mind. These are all laminar, usually very low Reynolds number, flows of limited extent. Many are related to rheological measuring devices, e.g. tube flow, flow between rotating disks, cones, cylinders or spheres, and squeeze flow; others to processing equipment, e.g. flow around spheres, in ducts of varying cross-section or in thin films and filaments. This section, not unexpectedly, is ideally suited as a starting point for chemical engineers, but may also appeal to students in many other disciplines.

All textbooks on fluid mechanics have to face the crucial questions of how tensor quantities are to be expressed and what knowledge of tensor operations is expected of the reader. Bird, Armstrong & Hassager follow closely in the path of Bird, Stewart & Lightfoot. They describe their notation in a long appendix, and base their manipulative demonstrations primarily on dyadic notation in a fixed Cartesian co-ordinate system. This is readily translatable into and from the co-ordinate-free symbolic notation associated with the schools of rational mechanics, and where necessary is expanded to include firstly the two orthogonal curvilinear co-ordinate systems based on cylinder and sphere and then secondly more general non-orthogonal curvilinear co-ordinate systems. (Having tried many ways, including body tensor notation, to retain elegance and generality as long as possible, I am now more than prepared to follow the path of BA&H out of BS&L. It may seem unnecessarily lengthy at some stages, but it corresponds best with the procedures that most solvers of flow problems have to go through in practice. Ideally, of course, facility in all representations should be achieved by mature workers and bigotry in these matters should be despised.)

Chapter 3 on flow phenomena represents some of the best reading in the book. In a limited number of well-illustrated pages (40) the characteristic observable differences between Newtonian and viscoelastic fluid flows are described and partly interpreted. Two of the most fascinating of these are rod climbing and die swell; here, I should have given more prominence to Joseph & Fossdick's satisfying analysis of the rod climbing phenomenon for slow flows (which is relegated to a footnote) than to Tanner's early and unreliable empirical prediction for die swell, but that is to find fault on a matter that is not at the heart of the book.

Chapter 4 introduces rheological equations of state by stealth and follows the historical path of considering one-by-one certain specific kinematic flow fields, for which the dynamics of motion is trivially understood. Thus steady and oscillating simple shear flow yield various viscosities and normal-stress differences while axisymmetric irrotational (called shear-free) flow yields an elongational stress function. Chapter 5 considers what are essentially engineering applications of the generalized Newtonian fluid model, thus relating primarily to simple shear flow, while chapter 6 provides a relatively concise account of the general linear viscoelastic fluid, which avoids any finite deformation complications. This is the point at which most elementary courses would end.

Chapters 7, 8 and 9 represent the most significant element in volume 1. The authors have contributed their full share of the original results that are reported here. Their intention has been to organize, tabulate and comment on a very wide range of proposals for rheological equations of state for viscoelastic liquids – indeed I believe they have attempted to cover all those that relate the stress tensor to the strain (or rate of strain) tensor directly, i.e. without recourse to additional structural variables. For pragmatic reasons they choose to base their development on a quasi-linear corotational model (chapter 7), i.e. to express differential or integral equations of state for fluid particles in a frame of reference rotating with the particles. This leads to Jaumann (time) derivatives and a great deal of space is spent demonstrating precisely how to move from the corotating frame into a fixed frame suitable for solving boundary-value problems. Having done this they then investigate the predictions of various quasi-linear models, namely the three-constant corotational Jeffery's model, the generalized corotational Maxwell model, and the Goddard–Miller equation.

Chapter 8 considers nonlinear corotational models, starting with the eight-constant Oldroyd model (one of the earliest, 1958, and still one of the most flexible, even if it is in theory very restricted), continuing with the general memory integral expansion and the retarded motion expansion and finally particularizing to simplified equations for steady-state flows derived from Rivlin. Chapter 9 introduces the codeformational formalism and so comes close to the approach of Oldroyd and Lodge. Much of chapters 7 and 8 is carried over into this new frame of reference, while its physical significance is explained. Those brought up on the all-embracing but highly formal approach of Truesdell, Coleman & Noll may miss any elaborate discussion of invariance and objectivity, and may think the less of this book for its absence. However, after a succession of textbooks on continuum mechanics that have been devoted in large measure to describing tensor analysis, frame invariance and material objectivity, it is a sign of progress that some knowledge of these can now be taken for granted.

Volume 2 is based on the chemical physicists approach, via kinetic theory, and seeks to derive rheological equations of state from the known long-chain-molecular nature of polymers. Chapters 10–13 consider dilute suspensions of proto-polymers – elastic dumbbell, rigid dumbbell, flexible chain and finally general bead–rod–spring models – having specified fluid–particle interactions, using a formalism that combines a continuum description of the suspending fluid with a statistical mechanical description of the suspended particles. Chapter 14 retreats into a full phase-space kinetic theory for both solvent and solute molecules in order to derive in a consistent fashion the relevant expressions for the stress tensor that are quoted without full justification in the earlier chapters. I found this chapter helpful and satisfying. Chapter 15 gives a careful, uncluttered account of various network models, emphasizing precisely what approximations and assumptions are made and where it is most reasonable to make them. Some comparison with experiments on concentrated solutions is included.

All that is presented in this volume is very well presented. Volume 2 is a unique text of great value. However, it must be said that it fails to give uniform coverage of all existing synthetic theories for dilute or concentrated suspension rheology. Brief mention only is made of the recent work of Batchelor, Leal and Hinch, or of earlier results of Bretherton, Brenner and Cox. To the extent that the rigid-dumbbell model is said to give a best fit to results for dilute polymer solutions, then surely many of the rigorous results for suspensions of ellipsoids, particularly those of large aspect ratio, should have been presented and compared with experiment.

One of the crucial assumptions in any of the suspension theories is that which specifies the interaction between the suspending fluid and the suspended particle: whether lumped or distributed, for all the theories in question it is always chosen to be linear in a local velocity difference. The point about the work based directly on Jeffery's original results for suspended ellipsoids is that the relevant stress tensor is, in any given realization for the flow, a well-defined and traditional continuum quantity with a local value that takes account of the finite extent of the suspended particles, and is continuous across interfaces between fluid and particle. The mean bulk stress is then obtained by simple statistical averaging according to conventional assumptions, while Brownian motion can be included in traditional fashion. The net result is a prescription for calculating the stress tensor which is, at least at first sight, quite different from that given in chapter 14, though they must in principle be compatible.

The virtues of the alternative approach not described by Bird, Hassager, Armstrong & Curtiss are most evident when considering the case of rigid biopolymers or flexible strings. BHA&C base all their results (p. 648) on variable scalar resistance coefficients for beads, whose magnitudes depend upon the extent of shielding, which implies that the drag will still always be parallel to the relative velocity. However, the results of Bretherton, Cox, Brenner, etc. specifically allow for non-isotropic resistance tensors for the lumped or distributed elements of a long molecule and make direct use of detailed and exact solutions of the Stokes equation. This is not to say that these other theories are any less complicated: they are however worthy of consideration by any seeking to make further progress in the field of predictive rheology.

Virtually no mention is made of the work of Freed & Edwards or de Gennes in connexion with entangled polymer melts. In that this other statistical mechanical work has not yet been taken anywhere near the stage of providing continuum rheological equations of state of the type discussed in chapters 7–9, this neglect is justified. Nevertheless, the approach may well prove equally successful in the end.

On the whole, the books are beautifully produced, and the printers have coped well with complicated mathematical equations. However, your reviewer is surprised that so many unbalanced and almost illegible flights of typesetting, such as

$$\left(\frac{1}{4\pi}\sin\theta\right)\left(\frac{4\pi}{J}e^{3\lambda(\kappa:\delta_R\delta_R)}\right),$$

reproduced from p. 531, should have got past the usually rock-like barrier of Professor Bird, a virtuoso of elegant and informative visual presentation. Perhaps it was in return for the lines of doggerel heading chapter 3:

A fluid that's macromolecular
Is really quite weird – in particular
The abnormal stresses
The fluid possesses
Give rise to effects quite spectacular.

J. R. A. Pearson